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|  | ANDERSON SERANGOON  JUNIOR COLLEGE | |
| **MATHEMATICS**  **H2 Mathematics Promotional Exam Paper (100 marks)** | | **9758**  **1 Oct 2021**  **3 hours** |
| Additional Material(s): List of Formulae (MF26) | | |

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| **READ THESE INSTRUCTIONS FIRST**  Write your name and class in the boxes above.  Please write clearly and use capital letters.  Write in dark blue or black pen. HB pencil may be used for graphs and diagrams only.  Do not use staples, paper clips, glue or correction fluid.  Answer **all** the questions and write your answers in this booklet.  Do not tear out any part of this booklet.  Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.  You are expected to use an approved graphing calculator.  Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.  All work must be handed in at the end of the examination. If you have used any additional paper, please insert them inside this booklet.  The number of marks is given in brackets [ ] at the end of each question or part question. |  | Question  number | Marks |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
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| 8 |  |
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| This document consists of 21 printed pages and 3 blank pages. |

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| **1** | It is given that . Show that , where *A* is a real constant to be determined. | [4] |
|  |  |  |
|  | **Solution** |  |
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|  | *A* = 3 |  |
|  |  |  |
| **2** | A graph with the equation  undergoes, in succession, the following transformations:  *A*: A translation of 1 unit in the direction of the negative *x*-axis.  *B*: A scaling parallel to the *x*-axis by a scale factor of .  *C*: A reflection in the *y*-axis.  Determine the equation of the resulting curve. | [4] |
|  |  |  |
|  | Solution |  |
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|  |  |  |
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| **3** | **(a)** Find . | [3] |
|  | **(b)** Find . | [3] |
|  |  |  |
|  | **Solution** |  |
|  | **(a)** |  |
|  |  |  |
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|  |  |  |
|  |  |  |
|  | **(b)** |  |
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| **4** | A curve *C* has equation |  |
|  | **(i)** Find  in terms of *x* and *y*. | [2] |
|  | **(ii)** Find the equation of the normal to the curve at the point *P*(2, 3). | [2] |
|  | **(iii)** Given that *C* meets the *y*-axis at the point *R* and the normal in **(ii)**meets the *y*-axis at the point *A*, find the area of triangle *APR* in the form , where *a* and *b* are integers to be determined. | [3] |
|  |  |  |
|  | **Solution** |  |
|  | **(i)** |  |
|  | Differentiate with respect to *x*: |  |
|  |  |  |
|  | **(ii)** Gradient of tangent to curve at *P*(2, 3)  =  Gradient of normal to curve at *P* = |  |
|  | Equation of normal at *P*: |  |
|  | **(iii)** When *x =* 0, normal cuts *y*-axis at *A*(0, 4);  *C*:  When *x =* 0, = 3 ⇒ y =  *C* meets *y*-axis at *R*(0, ). |  |
|  | Area of triangle *APR*  =  =  where *a* = 4, *b* = 3 |  |
|  |  |  |
| **5** | *y*  *x*  0.5    1  (4, *a*)  *y* = f(*x*) |  |
|  | The diagram above shows the graph of *y* = f(*x*). It has a maximum point at  (4, *a*), where *a* > 0, and meets the axes at (1, 0) and (0, 0.5). The curve has asymptotes with equations *y* = 0 and *x* =. |  |
|  | On separate diagrams, sketch the graphs of |  |
|  | **(a)**; | [3] |
|  | **(b)**, | [3] |
|  | stating the equation(s) of any asymptotes and where possible, the coordinates of any turning point(s) and axial intercept(s). |  |
|  |  |  |
|  | **Solution** |  |
|  | **(a)**  *y*  *x*  1  (4,)  *y* = f(*x*)  (, 0)  (0, 2)      *x* = 1 |  |
|  | **(b)**  *y*  *x*    (1, 0)    (4, 0)  *y* = 0  *x* = |  |
|  |  |  |
| **6** | The functions f and g are defined by    , . |  |
|  | 1. For this part of the question, it is given that. Show that the composite function fg exists. Hence find the range of fg. | [3] |
|  | 1. It is now given that is a real constant that is not necessary equals to 5. If the domain of f is restricted to , find in a similar form. | [4] |
|  |  |  |
|  | **Solution** |  |
|  | **(i)** |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  | **(ii)** |  |
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|  |  |  |
|  |  |  |
| **7** | **(a)** Find . | [1] |
|  | Hence find . | [2] |
|  | **(b)** Find. | [3] |
|  | **(c)** For , find the value of  in terms of *p*. | [3] |
|  |  |  |
|  | **Solution** |  |
|  | **(a)** |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  | **(b)** |  |
|  |  |  |
|  |  |  |
|  | **(c)** |  |
|  |  |  |
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| **8** | Relative to origin *O*, the points *A* and *B* have position vectors **a** and **b** respectively, where **a** and **b** are non-zero vectors. The point *C* is on *BA* produced such that  and *OC*  is perpendicular to *OB*. |  |
|  | 1. Find  in terms of **a** and **b**. | [1] |
|  | 1. Show that . | [2] |
|  | The point *P* is on the line *OB* such that it is the image of *B* in the line *OC*. |  |
|  | **(iii)** Find the area of triangle *PCB*. Leave your answer in the form of , where *k* is an exact real constant. | [3] |
|  | The point *F* is the foot of perpendicular of *P* to the line *AB*. |  |
|  | **(iv)** Given that , find the position vector of *F* in terms of **a** and **b**. | [4] |
|  |  |  |
|  | **Solution** |  |
|  | **(i)** By Ratio Theorem, |  |
|  |  |  |
|  | **(ii)** |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  | **(iii)** Area |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  | **OR** |  |
|  | Area |  |
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|  | **(iv)** Since *F* is on the line *AB*, |  |
|  | for some |  |
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| **9** | The curve *C* has equation    where *p*, *q* and *s* are non-zero constants.   1. It is given that *C* passes through the point and has a vertical asymptote . |  |
|  | **(i)** State the value of *s* and show that the value of *q* is 2. | [2] |
|  | **(ii)** It is given further that the line  is a tangent to *C* and it does not meet the curve again. Find the exact value of *p* if *p* is a negative real value. | [3] |
|  | **(b)** It is now given instead that. |  |
|  | **(i)** Sketch the curve *C*, showing clearly the coordinates of any turning point(s), equations of any asymptotes and the coordinates of any points of intersection with the axes. | [3] |
|  | **(ii)** Find the equation of the additional curve that needs to be added to the curve sketched in **(b)(i)** to determine the number of distinct real roots for the equation . | [2] |
|  |  |  |
|  | **Solution** |  |
|  | **(ai)** Since  is a vertical asymptote, |  |
|  |  |  |
|  |  |  |
|  | **(ii)** When *y* = 1, |  |
|  |  |  |
|  |  |  |
|  | Since  *y* = 1 is a tangent to *C*, |  |
|  | Discriminant = 0 |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  | **(bi)**              ( |  |
|  | **(bii)** |  |
|  | The equation of the additional curve is |  |
|  |  |  |
| **10** | **(a)**  **Fig. 1 Fig. 2**  8 cm  *x* cm  *x* cm  height  Fig. 1 shows the cylindrical-shaped water pipe, with negligible thickness and open on both ends, inscribed in a hemisphere with fixed radius 8 cm. The cross sectional view of the pipe and the hemisphere is shown in Fig. 2.  **(i)** If the diameter of the pipe is *x* cm, show that the curved surface area, *S*, of the pipe is  c. | [2] |
|  | **(ii)** It is given that as *x* varies, the maximum value of *S* occurs when the ratio of the diameter of the pipe to its height is . Find the exact value of *k* and the exact maximum value of *S*. | [6] |
|  | **(b)** A particle is moving on the curve with equation |  |
|  | where is the coordinate of the particle at time *t* relative to a fixed point *O*. The *x* and *y* values represent the horizontal and vertical displacements. |  |
|  | When the̶ ordinate of the particle is , the rate at which the̶ ordinate is decreasing with time *t* is 2 units per second. At this instant, find the exact rate at which the *x* - ordinate of the particle changes with time. | [4] |
|  |  |  |
|  |  |  |
|  | **Solution** |  |
|  | **(ai)** Height of cylinder =  = 2 |  |
|  | =  c (shown) |  |
|  | **(ii)**  =  = |  |
|  | For stationary point,  = 0 |  |
|  | (since *x* > 0) |  |
|  | *x* : 2= 1 : *k* |  |
|  | : 2 = 1 : *k* |  |
|  | *k* = 2 |  |
|  | First derivative test: |  |
|  | |  |  |  |  | | --- | --- | --- | --- | | *x* |  |  |  | |  |  | 0 |  | | Shape |  |  |  | |  |
|  | *S* is maximum when *x* = . |  |
|  | Second derivative test: |  |
|  | = |  |
|  | When   < 0 ⇒ S is maximum when . |  |
|  | Max *S* = |  |
|  | **(b)** |  |
|  |  |  |
|  | ⇒ |  |
|  | When =, 3*x* = sin=  *x* = |  |
|  |  |  |
|  | = |  |
|  | = |  |
|  | units/s |  |
|  |  |  |
| **11** | 1. Ivy took a $40 000 tuition fee loan for her 4-year university course that commences on 1st January 2021. The loan is interest-free during the period of study. Immediately after graduation, interest is charged at 4% per annum of the outstanding amount owe at the end of each year. The maximum loan repayment period is at most 15 years upon graduation. Ivy is planning to pay $550 every month upon graduation. |  |
|  | 1. Show that the amount she owes at the end of the *n* years after graduation is . | [3] |
|  | 1. Will she be able to finish repaying the loan by the end of 2030? Justify your answer clearly. | [2] |
|  | 1. Find the minimum monthly repayment Ivy should make if she intends to utilize fully the loan repayment period. | [2] |
|  | **(b)** To save for her tuition fee loan repayment, Ivy wishes to start a new savings plan on the first day of November 2021. In this plan, she needs to invest $200 into the account on the first day of each month. Every $200 invested earns a fixed interest of *d* % of $200 at the end of each month until a withdrawal is made from the account. The interest is added to the account and does not accumulate further interest. |  |
|  | 1. How much interest, in terms of *d*, will the first $200 deposited earn at the end of 2022? | [2] |
|  | 1. Find the least value of *d* such that the total amount in the account exceed $10 000 at the end of 36 months. | [3] |
|  |  |  |
|  | **Solution** |  |
|  | **(a)(i)**   |  |  |  | | --- | --- | --- | | Year / *n* | | Outstanding amount at the end of year | | 2025 | 1 |  | | 2026 | 2 |  | | 2027 | 3 |  | |  |  | … | |  |
|  | Amt owe at the end of *n* years = |  |
|  |  |  |
|  | **(ii)** At the end of 2030, *n* = 6 |  |
|  | Outstanding amount at end of 2030 |  |
|  | She will not be able to finish repaying the loan by the end of 2030. |  |
|  |  |  |
|  | **(iii)** Let *m* be the monthly loan repayment. |  |
|  | To utilise fully the loan repayment period, *n* = 15 |  |
|  |  |  |
|  |  |  |
|  | Minimum monthly repayment = $288.28 |  |
|  |  |  |
|  | **(b)(i)**   |  |  | | --- | --- | | *n* | Amount at end of month | | 1 |  | | 2 |  | | 3 |  | |  | .  . | | *n* |  | |  |
|  | At end of 2022, *n* = 14 |  |
|  |  |  |
|  |  |  |
|  | **(b)(ii)**   |  |  |  | | --- | --- | --- | | *n* | Amount at start of month | Amount at end of month | | 1 |  |  | | 2 |  |  | | 3 |  |  | | .. | … | … | | *n* | … |  | |  |
|  |  |  |
|  |  |  |
|  | Least *d* = 2.11 (3sf) |  |
|  |  |  |
| **12** | Relative to the origin *O*, a point *A* has position vector . The plane *p*1 has equation . |  |
|  | **(i)** Find the position vector of the foot of perpendicular from point *A* to the plane *p*1. | [4] |
|  | The line *l* has equation . |  |
|  | **(ii)** Find the acute angle between the plane *p*1 and the line *l*. | [2] |
|  | **(iii)** The point is equidistant from the plane *p*1 and the line *l*. Find the possible values of . | [4] |
|  | The plane *p*2 has equation , . |  |
|  | **(iv)** Show that the point  lies on both  *p*1 and *p*2. Hence find the vector equation of the line of intersection between the planes *p*1 and *p*2. | [3] |
|  |  |  |
|  | **Solution** |  |
|  | **(i)** Let *F* be the foot of perpendicular from *A* to the plane *p*1. |  |
|  |  |  |
|  | Since *F* lies on line *AF* |  |
|  | for some |  |
|  | As *F* is also on the plane *p*1, |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  | **(ii)** Let be the acute angle between the plane *p*1 and the line *l*. |  |
|  |  |  |
|  |  |  |
|  | **(iii)** Let point *D* and *E* be and respectively. |  |
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|  | **(iv)**  And  Hence *C* lies on both *p*1 and *p*2. |  |
|  |  |  |
|  | Eqn of the line of intersection is |  |

**End of Paper**